Further modeling and new results of active noise reduction using elasto-poroelastic panels

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Abstract

The paper presents further development in modeling of active elasto-poroelastic sandwich panels. In fact, a new design of a demi-sandwich panel is proposed and analysed. A numerical model of panel is implemented in *COMSOL Multiphysics* environment using the most fundamental but very flexible *Weak Form PDE Mode*. Various physical problems are modeled using Finite Element Method: the wave propagation in acoustic and poroelastic medium, the vibrations of elastic plate, the piezoelectric behavior of actuator. All these problems interact in the examined application of active panel. The presented results of FE analysis and some analytical solutions prove the necessity of modeling the panel's interaction with an acoustic medium. Again, confirmed is the fact that an active control is necessary for lower resonances while for the higher frequencies the passive reduction of vibroacoustic transmission performed by a well-designed poroelastic layer is sufficient.

1 Introduction

Recently, an exact modeling and analysis of a sandwich panel made up of two elastic faceplates and a poroelastic core has been presented [10]. The panel has been subjected to a harmonic excitation modeled as a directly applied uniform pressure. This time we examine a panel made up of a single elastic plate and a layer of poroelastic material but an important interaction of the panel with an acoustical medium (the air) is taken into account. Like previously, the panel is supposed to be an active noise absorber, so piezoelectric patches are fixed to the elastic faceplate forming a piezo-actuator. The actuator can be used to affect the bending vibrations of faceplate and in this way the vibroacoustic transmission through the panel should be controlled. We present the modeling of panel and the results of frequency analysis concerning the panel design and modeling, and finally, a numerical test of vibroacoustic control.

2 Comments on theoretical background

2.1 Biot theory of poroelasticity

An important component of the panel's assembly is a layer of porous material. Porous materials are quite often modeled in acoustics by using the so-called *fluid-equivalent* approach (see [1]). This is because for many porous materials in some applications the vibrations of skeleton can be neglected and then the so-called models of *porous materials with rigid frame* are valid. There are, however, many applications where the contribution of elastic frame vibrations is very significant. This is the very case of sandwich or demi-sandwich panels. Then, the simple fluid-equivalent modeling is no longer valid and instead a much more complicated *theory of poroelasticity* must be used. In this theory a *biphasic modeling* is applied: there is the so-called *solid phase* to describe the behaviour of elastic skeleton, and the *fluid phase* which pertains to the fluid in the pores.

The most frequently used is Biot isotropic theory of poroelasticity [5, 1]. In this modeling the both phases are isotropic. Moreover, the fluid is modeled as perfect (i.e. inviscid), but viscous forces, are taken into account though only when describing the interaction between the fluid and the frame.

There are two formulations of Biot's isotropic poroelasticity:

- the classical *displacement formulation* proposed by Biot where the unknowns are the solid and fluid phase displacements which means 6 degrees of freedom in every node of a 3-dimensional numerical model,
- the so-called *displacement-pressure formulation* where the dependent variables are the solid phase displacement and the fluid phase pressure. Therefore, there are only 4 degrees of freedom when modeling in 3D.

This latter formulation was presented by Attala et al. [2], and it is only valid for harmonic motion. Debergue et al. [7] discussed a very important subject of coupling and boundary conditions for this formulation. The slightly enhanced version of this formulation [3] was used by the authors of the present paper to model the poroelastic layer of sandwich panel.

2.2 Multiphysical character of modeling active elasto-poroelastic panels

The examined application of active elasto-poroelastic panels links several physical problems, namely:

- poroelasticity – to model poroelastic layer and its passive influence on the reduction of vibroacoustic transmission through the panel,
- acoustics – to model the propagation of acoustic wave and the coupling of acoustic medium with the panel,
- elasticity – to model the vibrations of the panel's elastic faceplate,
- piezoelectricity – to model the piezo-actuators and the active control of panel's vibrations.

Moreover, all these problems are strongly coupled. We surveyed weak, variational formulations of poroelasticity, elasticity, piezoelectricity, and acoustics to properly model these problems and couplings.

To model poroelastic material the enhanced weak integral form [3] of the mixed displacement-pressure formulation was used. This form simplifies the imposition of boundary conditions and coupling with elastic and acoustic media. Hints and other important information on convergence and finite element modeling of poroelasticity found in [8, 9, 6] were also considered.

In our modeling we used the most common form of the variational piezoelectric equations. It can be found, for example, in [4]. This paper provides also a wide survey on piezoelectric finite element modeling and discusses the problems concerning the electromechanical coupling, induced electric potential and actuation mechanisms.

3 Modeling of active elasto-poroelastic panel

3.1 Implementation in COMSOL Mutliphysics

To design and analyse the active elasto-poroelastic panel we chose *COMSOL Multiphysics* software – a modern tool for finite element modeling of multiphysics problems. Nevertheless, we did not used any of the advanced Aplication Modes provided with the software. We decided that in our model all considered problems, i.e.: poroelastic, acoustic, elastic, and piezoelectric were modeled in this environment by using one of the so-called *PDE Modes*, namely the *Weak Form, Subdomain, PDE Mode*. This is the most fundamental and laborious but at the same time, the most flexible and conscious approach. This approach allowed us also to apply properly and economically the coupling conditions between poroelastic material and other media. Previously, in [10] the so-called *General Form PDE Mode* was used to implement poroelastic material and this proved to be quite cumbersome when imposing coupling and boundary conditions.

A thorough validation of all the weak form implementations was carried out by using some analytical solutions (for multilayered media and rectangular elastic plate vibrations) as well as some of the *COMSOL Multiphysics* Application Modes (namely, the *Acoustics Application Mode*, the *Solid Stress-Strain Application Mode*, and the *Piezoelectric Effects Application Mode*).

3.2 The assembly of active elasto-poroelastic demi-sandwich panel

In [10] a model of sandwich panel made up of two elastic faceplates and a poroelastic core was investigated. This time the examined configuration might be called a demi-sandwich panel since it consists of:

- a single simply-supported elastic plate,
- a poroelastic layer glued to the lower side of the plate.

Moreover, the whole assembly of active panel includes piezoelectric patches fixed to the plate, and the modeling assumes that the poroelastic layer is coupled with an acoustic medium which is subjected to an acoustic pressure excitation and transfers this excitation onto the panel.

The in-plane dimensions of the panel are $80 \times 80\,\mathrm{mm}$ and the total thickness is $12.8\,\mathrm{mm}$ where $0.8\,\mathrm{mm}$ is the thickness of the plate and $12\,\mathrm{mm}$ stands for the thickness of poroelastic layer (see Figure 1). The elastic material of plate is alumiunium ($E_{\mathrm{e}} = 70 \cdot 10^9\,\mathrm{N/m^2}$, $\nu_{\mathrm{e}} = 0.33$, and $\varrho_{\mathrm{e}} = 2700\,\mathrm{kg/m^3}$) and the poroelastic layer is made of a polyurethane foam which properties are given in Table 1. The pores are filled with the air. The acoustic medium is a $80 \times 80 \times 38\,\mathrm{mm}$ waveguide of the air. In the cases where the elastic plate (or layer) without the poroelastic layer is considered the thickness of acoustic layer is augmented by the thickness of poroelastic layer and amount to $50\,\mathrm{mm}$.

porosity ϕ	tortuosity α_∞	flow resistivity σ [$\mathrm{N\,s/m^4}$]	characteristic dimensions of pores Λ [m]	Λ' [m]
0.97	2.52	87000	$37 \cdot 10^{-6}$	$119 \cdot 10^{-6}$

elastic properties of the material of skeleton: density ϱ_{s} [$\mathrm{kg/m^3}$]	Young modulus E_{s} [$\mathrm{N/m^2}$]	Poisson coefficient ν_{s}
1033	$143000 \cdot (1 + \mathrm{j}\,0.055)$	0.3

Table 1: Poroelastic properties of polyurethane foam

In the complete assembly of panel piezoelectric transducers (0.5 mm thick) are glued to the upper surface of the elastic plate. They are through-thickness polarized so they stretch significantly in plane when a voltage is applied inducing in this way a bending deformation of the plate. The piezoelectric material of patches is the transversely isotropic ceramic PZT4.

Two configurations of transducers were checked:

- one square patch 30×30 mm in the centre of the upper side of plate,
- four square patches of 10×10 mm fixed symmetrically around the centre of plate.

In the latter configuration we assume that the actuators are to be excited simultaneously by the same voltage so they act in the same way on the plate. This configuration gave much better performance (bigger deflection of the center of the plate and better deformation shape) so it was used in the further analysis.

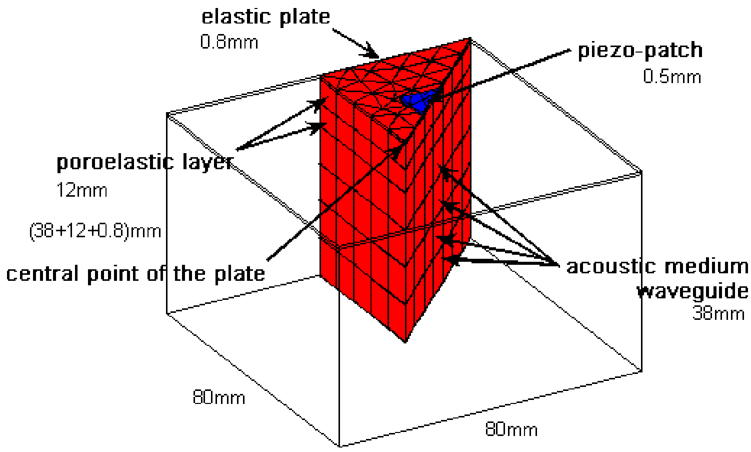

Figure 1: Dimensions of the system composed of the active elasto-poroelastic panel and acoustic medium waveguide, and the finite element mesh of the 1/8-slice model (a different, brick element mesh was used for the complete model of system composed of the elastic plate and 50 mm thick acoustic medium)

4 FE modeling and analysis

4.1 Results of the frequency analysis

Several numerical tests concerning the modeling of some parts or the whole assembly of the panel were carried out. A frequency response in a wide spectrum (up to 4500 Hz) was analysed. In some of the tests the panel or (only) the elastic plate is coupled with the acoustic medium (the air). Some of the tests were matched with analytical solutions.

Plate tests. Figure 2 presents results of the modeling of elastic plate and its coupling with the acoustic medium. The frequency response analyses for the following problems were performed:

- the FE model of plate coupled with the acoustic medium (the air) subjected to a uniform acoustic pressure excitation,
- the plate subjected (directly) to the uniform pressure (analytical and numerical solutions),
- the plate subjected to a concentrated force applied at the point $(\frac{80\,\mathrm{mm}}{4}, \frac{80\,\mathrm{mm}}{4})$ – the reason for this analytical frequency response solution is to reveal all the resonances for all the natural frequencies present in the considered spectrum.

In all the analyses the computed response was the maximum absolute value of the normal displacements of the plate divided by the similar result obtained for the static case; for the problems with the uniform pressure excitation (curves (a),(b), and (c) in Figure 2) the static solution was in fact identical. The presented results show that it is very important to take into consideration the acoustic-elastic coupling (rather than applying the acoustic pressure directly) since a new "*coupling*" resonance appears at 1735 Hz (curve (a)).

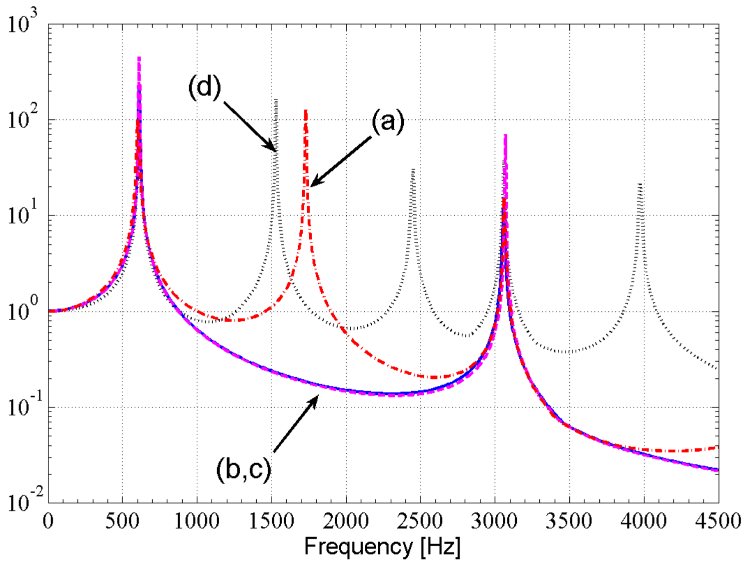

Figure 2: Results of the frequency analysis of: **(a)** the 3-dimensional FE model of the plate coupled with the acoustic medium subjected to the excitation of uniform pressure inducing an initially-plane acoustic wave in the waveguide in the direction perpendicular to the plate, **(b,c)** the plate subjected (directly) to the uniform pressure – the analytical and FE solution (the curves overlap); **(d)** the plate subjected to a concentrated force (analytical solution).

The numerical model of plate was 3-dimensional since the complete assembly of active panel consists also of some piezo-actuators which exert an influence on the plate and in this case only 3-dimensional medeling of piezoelectric effects and the plate may be exact. The 3-dimensional FE model of the plate was chosen to be a regular $5\times5\times1$ brick element mesh with cubic shape functions to approximate all the displacement fields. A very good conformity of at least 13 natural frequencies is achieved for this FE model – to check this an eigenvalue problem was solved. This is also confirmed by the frequency analysis – see curves (b,c) in Figure 2. We also checked that for a much more denser meshes but with only quadratic shape functions this conformity is inferior.

The acoustic medium layer (the air) was modeled as a regular $5\times5\times5$ brick element mesh with quadratic shape functions used to approximate the acoustic pressure field. The thickness of the whole layer was 50 mm and for the highest frequency considered (i.e. 4500 Hz) the acoustic wavelength equals 76 mm.

Multilayer tests. Figure 3 shows results obtained analytically for 1-dimensional problems of wave propagation in multilayered media – two configurations were examined:

- acoustic layer of thickness 50 mm coupled with 0.8 mm layer of aluminium,
- 38 mm acoustic layer + 12 mm poroelastic layer + 0.8 mm layer of aluminium.

Let us notice here that the total thickness for both configurations is the same. The excitation was harmonic unit acoustic pressure applied on the acoustic layer and the results are the amplitudes of displacements of aluminum layer (1 m was used as the reference). They are compared with the similar results obtained from the FE analysis of the aluminium plate coupled with the acoustic medium. We have only one – coupling resonance in the case of 1D multilayered systems. Now, the coupling genesis of the resonance at 1735 Hz is excellently confirmed. Moreover, damping properties of poroelastic layer reveal thanks to this simple 1-dimensional analysis.

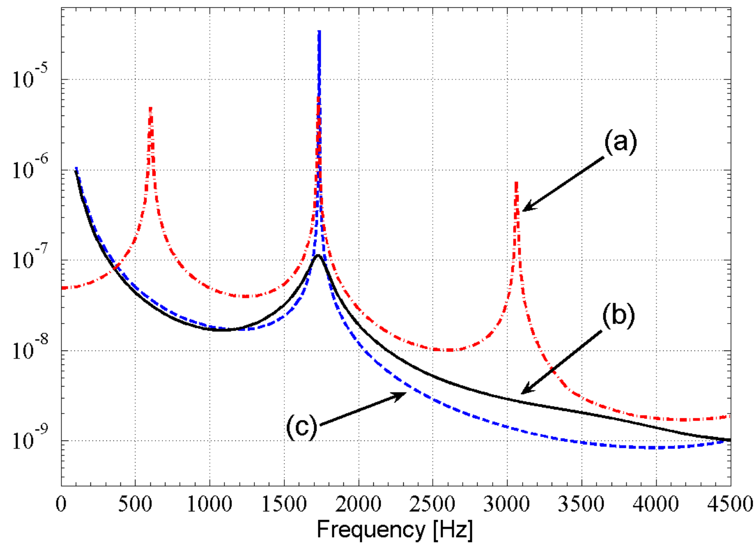

Figure 3: **(a)** the same results as presented in Figure 2 (curve (a)) but here 1 m was used as the reference (the pressure was 1 Pa); **(b)** results of the 1-dimensional (analytical) analysis of two-layered medium composed of the air (50 mm) and aluminium (0.8 mm); **(c)** results of the 1-dimensional (analytical) analysis of three-layered medium composed of the air (38 mm), poroelastic material (12 mm), and aluminium (0.8 mm).

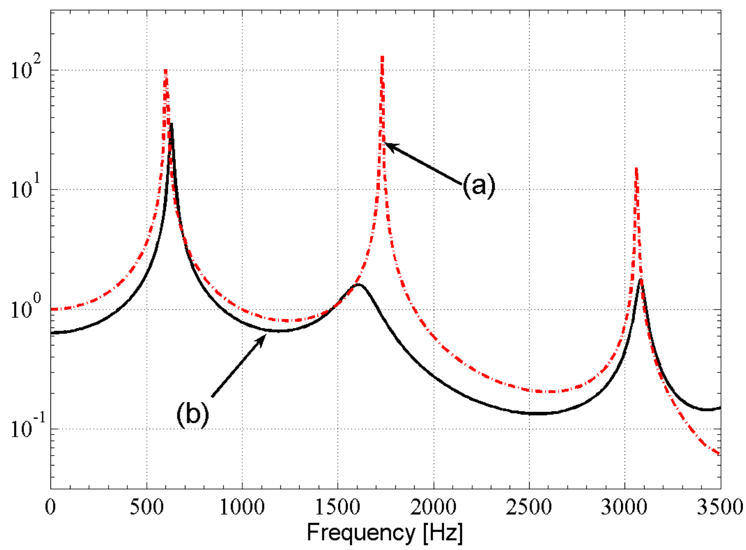

Figure 4: **(a)** the same curve as in Figure 2; **(b)** results of the FE analysis of the complete assembly of panel, i.e. the simply-supported plate (with passive piezo-patches) composed with the poroelastic layer coupled with the acoustic medium (a limited 1/8-slice model was used).

Panel test. Figure 4 presents the frequency analysis for the plate linked with the poroelastic layer coupled with the acoustic medium. This time the piezoelectric patches are present but they are passive (i.e. the only effect is some locally added mass and stiffness). The results (the maximal amplitudes of the elastic plate displacements) are compared with the results (presented already in Figures 2 and 3) of the FE analysis of the plate coupled directly with the acoustic medium. The static solution for this latter problem is the reference value. The total thickness is the same for both configurations (providing that the thickness of piezoelectric patches is not considered). The resonances for the active elasto-poroelastic panel are slightly shifted (because of the locally added stiffness and mass of piezoelectric patches). The most important observation is that the coupling resonance (now app. 1600 Hz) and the higher resonance of simply-supported plate (now app. 3080 Hz) are attenuated when the poroelactic layer is present. On the contrary, the low frequency resonance of plate (first eigenfrequency, now app. 630 Hz) is fully manifested. And for this lower frequency an active control of vibrations is necessary.

We must admit that because of a considerable numerical cost only 1/8 of the active elasto-poroelastic panel was modeled (see Figure 1) taking full advantage of the symmetries. This is not quite rightful approach (since some antisymmetric modes are not represented); however, in the case of uniform acoustical pressure excitation this simplification seems to be acceptable. Figure 1 presents the FE mesh of this limited model: it is composed of parallelepiped elements with triangular base. Now, for the poroelastic layer and acoustic medium as well as for the elastic plate and the piezoelectric patch the quadratic shape functions were used for all the dependent variables. The plate and the piezo-patch are very thin and they were discretized with one layer of elements, for the 38 mm layer of acoustic medium four layers of mesh elements were used, whereas for the 12 mm layer of poroelastic material two layers of elements. This seems to ensure a sufficient convergence since this time the highest computational frequency was 3500 Hz and for this frequency the wavelength in the air is 98 mm, while the shortest wavelength in the poroelastic medium is the shear wave length equal to 12 mm.

4.2 Results of the active control analysis

The concept of vibroacoustic control using a sandwich panel was presented in [10]. This concept is valid in the present application where an incident acoustic wave propagates through the acoustic medium waveguide onto the panel's poroelastic layer. The wave may be attenuated by the poroelastic layer and partially reflected and transmitted through the panel.In the precious Section we have shown that for higher frequencies the passive reduction of vibroacoustic transmission is sufficient: the coupling resonance and the plate resonance close to the higher eigenfrequency are well attenuated (see Figure 4). Nevertheless, the resonance vibrations closed to the first eigenfrequency of plate are not attenuated and this is the task for the piezoelectric actuators which are used to counteract the panel vibrations so that the normal velocity of the plate is minimized which means the reduction of transmitted wave. On the elastic faceplate a piezoelectric sensor (not modeled here) can also be planted, so that the velocity can be measured and thus the transmission reduction process can be controlled. In the analysis of vibroacoustic transmission reduction we assume that the acoustic wave is harmonic. In practice, we can control transient or multi-frequency vibrations applying an appropriate signal to the actuator. This harmonic analysis, however, may also be used to control some predominant components of vibration spectrum, and since the problem is modeled linearly, we can superpose the results obtained for different frequencies.

Let us remind that the complete system considered here is composed of the active elasto-poroelastic panel and the acoustic medium waveguide. For the purpose of vibroacoustic control the system analysis for the two following cases must be performed:

- the unit acoustic pressure excites the panel through the acoustic domain,
- the unit voltage is applied onto the piezo-actuators.

In the analyses we compute the biggest deflection of the panel's elastic plate. In both cases, the excitation is harmonic, and the panel deflections are from steady-state response for the same harmonic frequency. Now, since the analysis is linear we can use the superposition principle to compute the deflexion for arbitrary values of pressure and voltage applied simultaneously. For the considered frequencies and for this sort of excitations the biggest deflections are in the centre of elastic plate. Therefore, a low transmission means small deflection (velocity) of the central point of elastic plate. We can control this deflexion by means of the piezoelectric actuators by applying an adequate voltage. This voltage we calculate by zeroing the central deflection obtained from the superposition of the both cases.

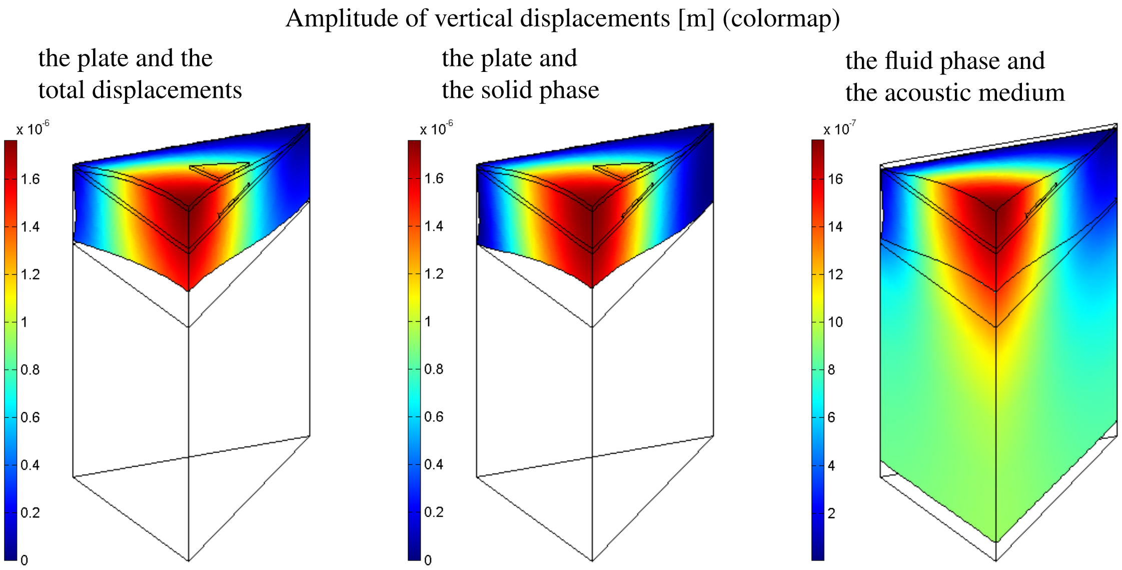

Figure 5: Uncontrolled vibrations for the unit acoustic pressure excitation (frequency 630 Hz)

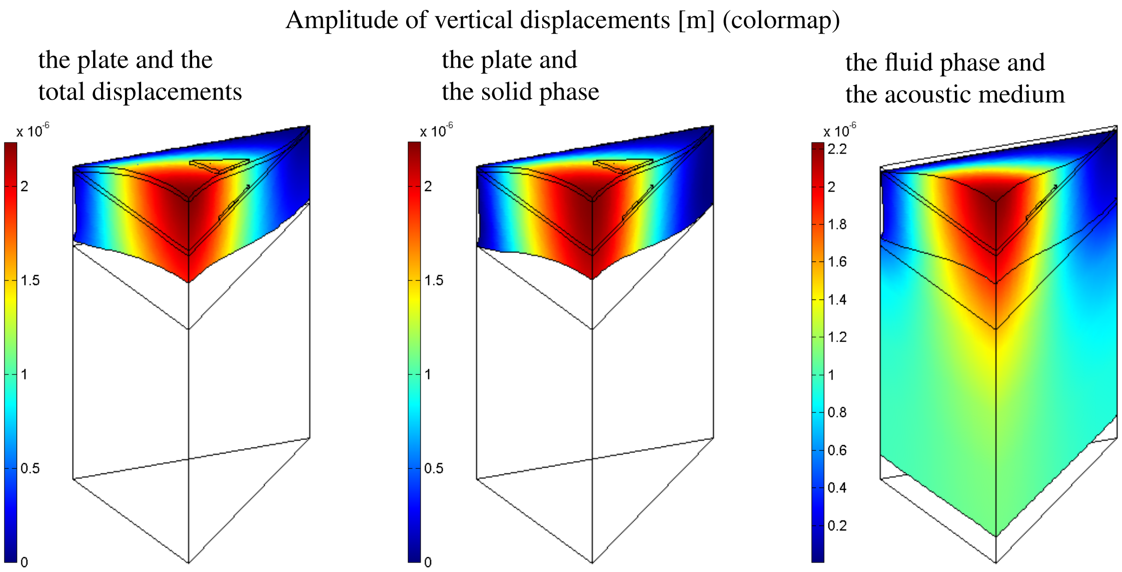

Figure 6: Vibrations for the unit voltage excitation (frequency 630 Hz)

Figures 5 and 6 shows the two steps of the vibroacoustic control analysis performed for the first resonance frequency (630 Hz). The deformations and amplitudes of vertical displacements are shown for the three following cases:

- the displacements of elastic faceplate and of the solid phase of poroelastic layer,
- the displacements of elastic faceplate and of the fluid phase of poroelastic layer,
- the displacements of the fluid phase of poroelastic layer and of the acoustic medium.

One should be aware that for all the plots the same deformation scaling factor was used. Figure 5 showing the uncontrolled vibrations of system confirms that for this resonance frequency there is a very significant vibroacoustic transmission through the panel. One should notice that the amplitudes of the plate and solid phase displacements are similar with the amplitudes of the fluid phase and acoustic medium displacements.

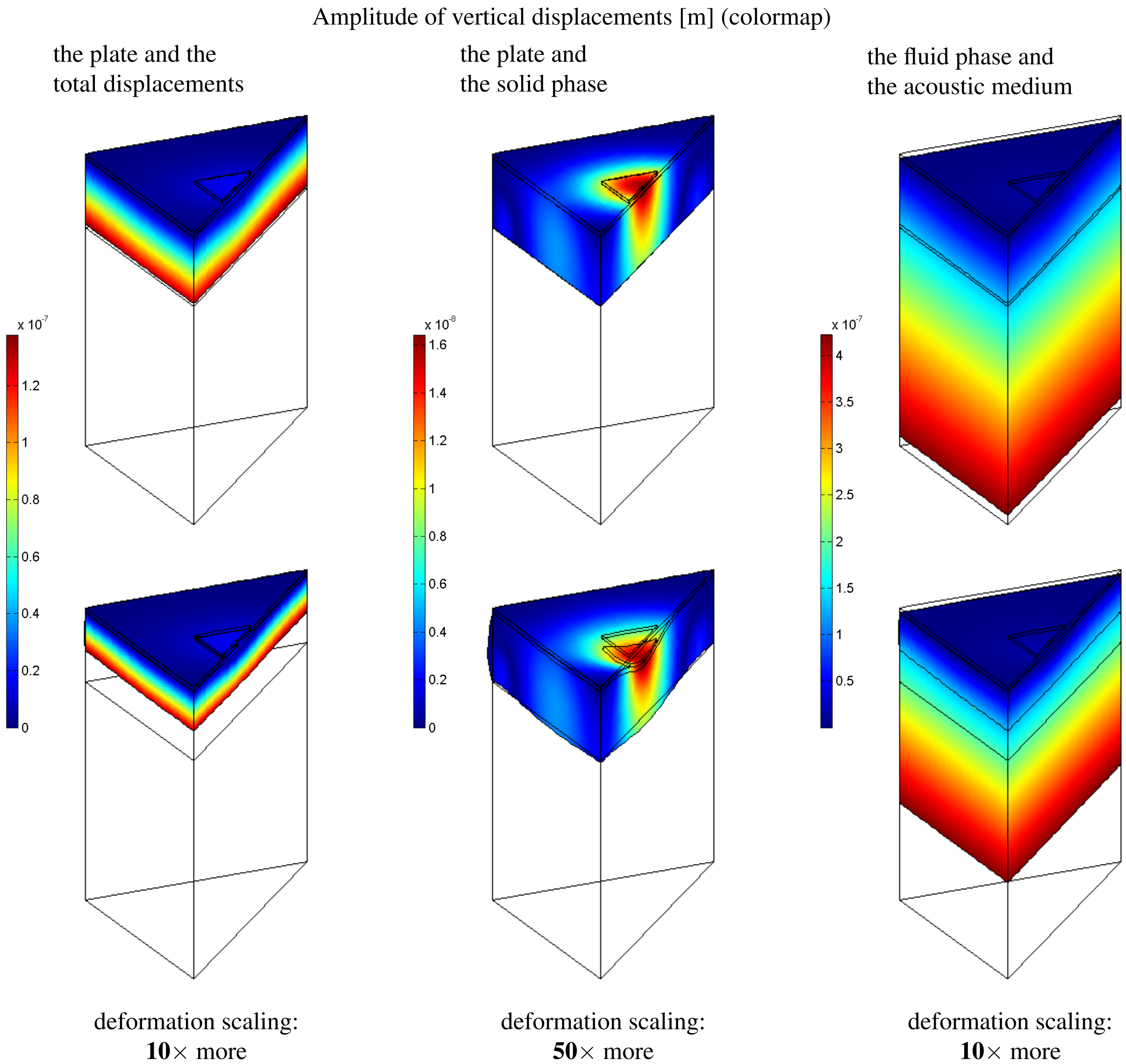

Figure 7: Vibroacoustic transmission reduced: the controlled vibrations for the simultaneous excitation with the unit pressure and adequate voltage (frequency 630 Hz). The corresponding plots shown in the upper and lower row differ only in the deformation scaling.

The same sort of deformations and amplitudes are presented in Figure 7 for the system under the simultaneous excitation with the unit acoustic pressure and the adequate voltage (computed thanks to the previous analyses). This time the deformations and vertical displacement amplitudes for the plate and the solid phase of poroelastic layer are significantly smaller. Even the fluid phase and (consequently) the total displacements of panel are smaller than in the case of uncontrolled vibrations. Therefore, the plots are presented in two versions regarding the magnitude of deformation scaling:

- (upper row) the same deformation scaling as in Figure 5,
- (lower row) the deformation scaling factor is 10 or even 50 times bigger than the one used for plots in the upper row.

We are certainly allowed to say that the results presented graphically in Figure 7 confirm a very good reduction of vibroacoustic transmission achieved with the help of the piezo-actuators.

5 Conclusions

A numerical FE design of a prototype model of the active demi-sandwich panel made up of an elastic plate with piezo-actuators (for active control of vibroacoustic transmission) and a poroelastic layer (designed for passive control) was presented. The results obtained from some frequency anlyses allow to draw the following conclusions:

- the modeling of interaction between the panel and the air is important because some acoustic-elastic coupling resonances may occur,
- the poroelastic layer should be designed to attenuate the higher frequency resonances,
- the lower frequency resonance(s) – especially around the first eigenfrequency of the panel's elastic plate – require an active attenuation with the help of a piezo-actuator.

Moreover, these results together with the analysis of active vibracoustic control permit to conclude that the proposed model of demi-sandwich panel should work properly for a wide frequency range as an efficient active/passive reducer of vibroacoustic transmission. Nevertheless, further and more advanced design consisting in some parametric survey and configuration optimization must be performed. A special attention must be paid on:

- a proper choice of poroelastic material and its thickness (for passive attenuation),
- an optimal design of piezoelectric actuator and its localization (for active control).

Additional requirements are obvious: the panel should be light and thin, the piezoelectric actuator – small.

We admit that some limitations were encountered when modeling the whole assembly of panel. Nevertheless, we may say that we are now able to carry out an exact 3-dimensional modeling of an active elasto-poroelastic panel immersed in an acoustic medium.

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